$$z = f(x,y) = 3x^2y + 4y^3 - 3x^2 - 12y^2 + 1 \qquad D = R^2$$

Find all local extrema on R^2.

$$f'_x(x,y) = 6xy - 6x \qquad f'_y(x,y) = 3x^2 + 12y^2 - 24y$$

$$f''_{xx}(x,y) = 6y - 6, \; f''_{yx}(x,y) = 6x \quad (f''_{xy} = 6x) \; f''_{yy}(x,y) = 24y - 24$$

$$\Delta(x,y) = \begin{vmatrix} 6y-6 & 6x \\ 6x & 24y-24 \end{vmatrix} = (6y-6)(24y-24) - 36x^2$$

<u>Stationary points.</u> $\begin{cases} f'_x(x,y) = 0 \\ f'_y(x,y) = 0 \end{cases} \rightsquigarrow \begin{cases} 6x(y-1) = 0 \\ 3x^2 + 12y^2 - 24y = 0 \end{cases}$

a) $x = 0$ $\rightsquigarrow 12y^2 - 24y = 0 \rightsquigarrow y = 0,\ y = 2 \rightsquigarrow (0,0),\ (0,2)$

b) $x \neq 0 \Downarrow y = 1 \rightsquigarrow 3x^2 - 12 = 0 \rightsquigarrow x = -2,\ x = 2 \rightsquigarrow (-2,1),\ (2,1)$

4 stationary points $\{$ $(0,0),\ (0,2)$, $(-2,1),\ (2,1)$

<u>Decision about local extremum.</u>

$\Delta(0,0) = (-6)\cdot(-24) > 0,\ f''_{xx}(0,0) = -6 < 0 \rightsquigarrow$ local max.

$\Delta(0,2) = 6 \cdot 24 > 0,\quad f''_{xx}(0,2) = +6 > 0 \rightsquigarrow$ local min.

$\Delta(-2,1) = -36 \cdot (-2)^2 < 0 \rightsquigarrow$ saddle point

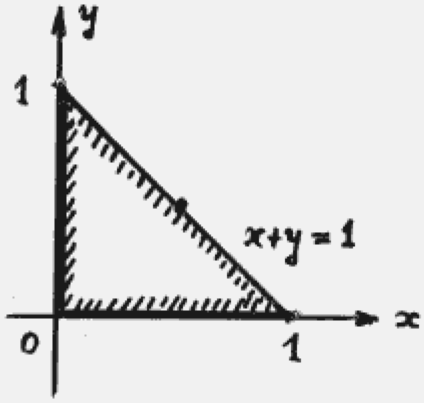
$\Delta(2,1) = -36 \cdot 2^2 < 0 \rightsquigarrow$ saddle point.

Beispiel

Man ermittele die absoluten Extrema der Funktion

$$f(x,y) = (10x-1)(10y-1)$$

auf der Menge D:



$f'_x(x,y) = 10(10y-1) = 0$

$f'_y(x,y) = 10(10x-1) = 0$

stationärer Punkt $(1/10, 1/10)$

$\Delta\left(\frac{1}{10}, \frac{1}{10}\right) = -10000 < 0$

$\rightsquigarrow$ kein lok. Extremum

Untersuchung auf dem Rand von D:

(3 Randstücke)

a) $x=0 \quad 0 \le y \le 1$. $f(x,y) = f(0,y) = -10y + 1 = \varphi(y)$.

Größter Wert von φ bei $y=0 \quad \varphi(0) = 1$.

Kleinster Wert von φ bei $y=1 \quad \varphi(1) = -9$.

b) $y=0 \quad 0 \le x \le 1$. $f(x,y) = f(x,0) = -10x+1 = \psi(x)$.

Größter Wert von ψ bei $x=0 \quad \psi(0) = 1$

Kleinster Wert von ψ bei $x=1 \quad \psi(1) = -9$.

c) Geradenstück $y = 1-x$, $0 \le x \le 1$.

$$f(x,y) = f(x,1-x) = (10x-1)(9-10x) = \omega(x).$$

$\omega'(x) = -200x + 100 = 0 \rightsquigarrow x = \frac{1}{2} \rightsquigarrow y = \frac{1}{2}$

$\omega\left(\frac{1}{2}\right) = 16$

$f(0,0) = 1 \qquad f(1,0) = \psi(1) = -9 \qquad f(0,1) = \varphi(1) = -9 \qquad f\left(\frac{1}{2}, \frac{1}{2}\right) = \omega\left(\frac{1}{2}\right)$

absolute Minima

absolutes Max $(=16)$