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1. Übung

1) Beschleunigte Bewegung

$\ddot{x}(t) = -g\, e^{-\gamma t}$

$\dot{x}(t) = \frac{g}{\gamma} e^{-\gamma t} + v_0 = \int_{t_0}^{t} \ddot{x}(t')\, dt'$ $\qquad c_1 = -\frac{g}{\gamma}$

~~$x(t) = -\frac{g}{\gamma} e^{-\gamma t} + v_0 t + x_0$~~ $\qquad \dot{x}(t=0) = v_0$

$\dot{x}(t) = \int_{t_0}^{t} \ddot{x}(t')\, dt' = \frac{g}{\gamma} e^{-\gamma t} - \frac{g}{\gamma}$ $\qquad v_0 = 0$, $x_0 = 600\,m$

$x(t) = -\frac{g}{\gamma^2} e^{-\gamma t} - \frac{g}{\gamma} t + \frac{g}{\gamma^2}$

! $\dot{x}(t=0) = 0 \quad \to \quad \dot{x}(t) = \int \ddot{x}(t')\, dt' = \underbrace{\frac{g}{\gamma} e^{-\gamma t} + c_1}_{=0}$

$\Rightarrow c_1 = -\frac{g}{\gamma}$

$x(t) = -\frac{g}{\gamma} e^{-\gamma t} - \frac{g}{\gamma} t + \frac{g}{\gamma^2} + 600\,m$

a) Fallzeit:

$x(t_F) = 0$ $\qquad$ Näherung $\quad e^{-\gamma t_F} \approx 0$

$= -\frac{g}{\gamma} t_F + \frac{g}{\gamma^2} + 600\,m$

$\Leftrightarrow t_F = \frac{1}{\gamma} + 600\,m \,\frac{\gamma}{g} = \underline{\underline{61\,s}}$

b) Aufprallgeschw.

$v(t) = \dot{x}(t) = \frac{g}{\gamma} e^{-\gamma t} - \frac{g}{\gamma}$ $\qquad e^{-\gamma t}$

$\approx -\underline{\underline{10\,\frac{m}{s}}}$

c) 98% v. Geschw.

$v(t_u) = 0{,}98 \cdot v(t_F) \approx 0{,}98 \cdot \frac{g}{\gamma}$

$:= \frac{g}{\gamma}(1 - e^{-\gamma t_u})$

$\Rightarrow e^{-\gamma t} = 0{,}02$

$\gamma t_u = \ln(50)$

$\underline{\underline{t_u = \frac{1}{\gamma} \ln 50 = 4\,s}}$

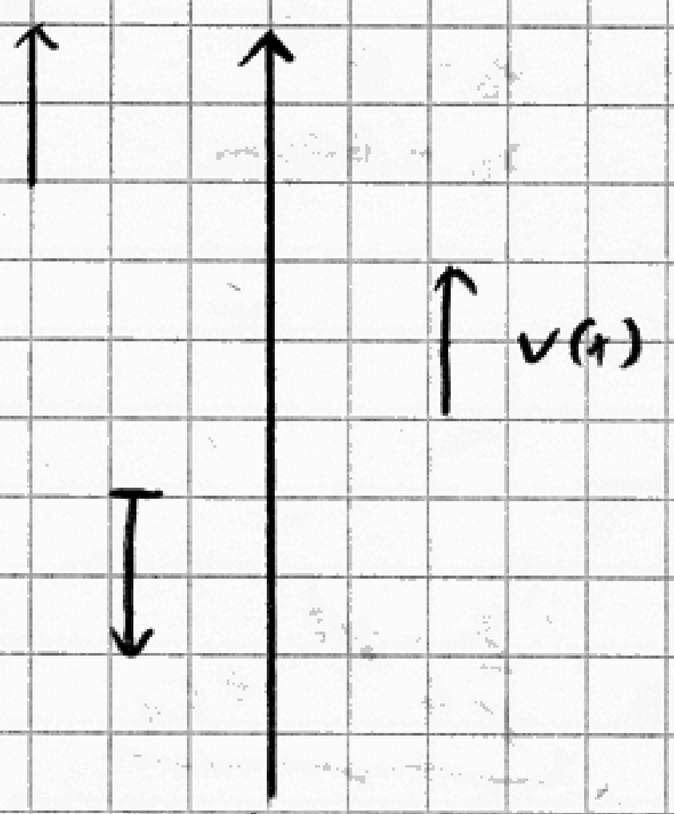
2) Methode d. Variablentrennung

v_0

$\ddot r(t) = \frac{\alpha}{r^2(t)}$

$r(t=0) = R$

$\ddot r(t=0) = -g$



x_1 $\quad \frac{\alpha}{r^2(t)} = \frac{1}{2}\frac{dv^2}{dr}$

$\int_{r_0}^{r} \frac{\alpha}{r^2(t)}\, dr = \frac{1}{2}\int_{v_0}^{v} dv^2$

$= \frac{1}{2}(v^2 - v_0^2)$

$\frac{1}{2}(v^2 - v_0^2) = \frac{gR^2}{r} - \frac{gR^2}{R}$

x_2 $\quad \frac{\partial v^2}{\partial r} = 2v\frac{\partial v}{\partial r}$

a) BWGL

$\ddot r(t) = \dot v(t) = \frac{d}{dt} v[r(t)]$

$= \frac{\partial v}{\partial r}\underbrace{\frac{dr}{dt}}_{v} = \frac{1}{2}\frac{\partial v^2}{dr}$ $\quad x_2$

$= \frac{\alpha}{r^2(t)} = \frac{1}{2}\frac{\partial v^2}{\partial r}$ $\quad x_1$

α: $\ddot r(t=0) = \frac{\alpha}{r^2(t=0)} = -g$

$\alpha = -gR^2$

$\frac{1}{2}(v^2 - v_0^2) = \int_R^r -\frac{gR^2}{\tilde r^2}\, d\tilde r$

$= \frac{gR^2}{r} - \frac{gR^2}{R}$

$\rightarrow v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{R}\right) + v_0^2$

b), c) Fluchttrajektorie

$v^2(r \to \infty) = 0$

$v^2(t) = v^2[r(t)] = 0 = 2\underbrace{\frac{gR^2}{r}}_{0} - 2gR + v_0^2$

c) $\quad v^2(t) = 2g\frac{R^2}{r(t)}$

$\frac{dr}{dt} = \sqrt{2g}\,\frac{R}{\sqrt{r(t)}} \quad \rightarrow \quad \sqrt{r}\, dr = \sqrt{2g}\, R\, dt$

$\int_R^r \sqrt{r}\, dr = r^{3/2} - R^{3/2}$

$\int_{t_0}^{t} \sqrt{2g}\, R\, dt = \sqrt{2g}\, R\,(t - \underbrace{t_0}_{0})$

$$r^{3/2} = \frac{3}{2}\sqrt{2g}\,Rt + R^{3/2}$$

$$r(t) = \left[\frac{3}{2}\sqrt{2g}\,Rt + R^{3/2}\right]^{2/3}$$

3. Harmonischer Oszillator

$$\ddot{x} + \omega^2 x = 0 \qquad \text{Ansatz } e^{\lambda t} = x$$

$$e^{\lambda t}\lambda^2 + \omega^2 e^{\lambda t} = 0$$

$$\lambda^2 + \omega^2 \Rightarrow \lambda_{1/2} = \pm\sqrt{-\omega^2} \qquad \lambda_1 = -i\omega \quad , \lambda_2 = i\omega$$

$$x_{1/2} = e^{\pm i\omega t}$$

$$\frac{d^2}{dt^2}\left(Ae^{i\omega t} + Be^{-i\omega t}\right) + \omega^2\left(Ae^{i\omega t} + Be^{-i\omega t}\right) = 0 \qquad \text{ist wahr}$$

$\Rightarrow$ ist auch Lösung

$x(t=0) = x_0$ $\qquad x(t) = Ae^{i\omega t} + Be^{-i\omega t}$

$v(t=0) = 0$ $\qquad x_0 = A + B$ $\qquad x_0 = 2A \Rightarrow A = \frac{x_0}{2} = B$

$$\dot{x}(t) = i\omega Ae^{i\omega t} + (-i\omega Be^{-i\omega t})$$

$$= v_0$$

$$= i\omega\left(Ae^{i\omega t} - Be^{-i\omega t}\right)$$

$$v(t=0) = i\omega(A - B) \Rightarrow A = B$$

$x(t=0) \neq 0$ $\qquad A = \frac{v_0}{2i\omega}$

inhomogener Fall

$$\ddot{x} + \omega^2 x = ct$$

$$\ddot{x} + \omega^2\underbrace{\left(x - \frac{c}{\omega^2}t\right)}_{y} = 0$$

$$\ddot{y} + \omega^2 y = 0 \quad \rightarrow \quad x_{1/2} = e^{\lambda_{1/2} t} - \frac{c}{\omega^2}t$$

$$\lambda = \pm i\omega$$

$$\ddot{x} + \omega^2 x = ct$$

$$x(t_0) = x_0$$

$$v(t_0) = v_0$$

$$t_0 = 0$$

partikulární

$$x = A x_1^{h} + B x_2^{h} + x_3^{(p)}$$

$$x_3^{(p)} = \frac{c}{\omega^2} t$$